u-substitution is backwards chain rule

Motivating example

	Example application of chain rule	Example application of u-substituion
1.	$f(x) = \sin\left(\underbrace{x^2+4x+3}_{\substack{u \\ \text{"pretend } x\text{"}}}\right)$	$f(x) = \sin(x^2+4x+3) + C$
2.	$f(x) = \sin(u)$	$f(x) = \sin(u) + C$
3.	$f'(x) = \cos(u) \cdot \_\_\_$	$f(x) = \int \cos(u)\, \mathrm{d}u$
4.	$f'(x) = \cos\left(\underbrace{x^2+4x+3}_{\substack{u \\ \text{"pretend } x\text{"}}}\right) \cdot \left(\underbrace{2x+4}_{\frac{\mathrm{d}u}{\mathrm{d}x}}\right)$	$f(x) = \int \cos\left(\underbrace{x^2+4x+3}_{\substack{u \\ \text{"pretend } x\text{"}}}\right) \cdot \left(\underbrace{2x+4}_{\frac{\mathrm{d}u}{\mathrm{d}x}}\right) \mathrm{d}x$
5.		Let $u = x^2+4x+3$, $\frac{\mathrm{d}u}{\mathrm{d}x} = 2x+4$
6.		$f(x) = \int \cos(x^2+4x+3) \cdot (2x+4)\, \mathrm{d}x$

Steps

1. Identify a **portion of the integrand that has a derivative that looks like some other stuff in the integrand**. Let this expression be u.
   a. If the choice of u is not obvious, try working through the following list:

Type of function	Have differentiation formula?	Have antiderivative formula?
Logarithms	Yes	No
Inverse trigonometric functions	Yes	No
Power and **p**olynomial functions	Yes	Yes
Exponential functions	Yes	Yes
Trigonometric functions	Yes	Yes

   b. If the earliest row in the above table that has a corresponding expression in the integrand describes more than one expression in the integrand, give higher priority to identifying as u expressions that are "inside" complicated operations (e.g. inside a power, inside a denominator, inside a cosine, etc.)
2. Compute $\mathrm{d}u = u'(x)\,\mathrm{d}x$. If algebraic manipulation is unreliable, explicitly isolate $\mathrm{d}x = \frac{\mathrm{d}u}{u'(x)}$.
3. If integral is definite, compute each bound in u (e.g. for $x = a$, write $u(a) = \cdots$).
4. Substitute u and $\mathrm{d}u$ into the integral (replacing the bounds if the integral is definite).
5. Attempt to integrate the resulting integral.
6. If integration succeeds and integral is indefinite, replace u with its expression in terms of x.